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Double Dispersion effects on free convection along a vertical Wavy Surface in Porous Media with Variable Properties

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Abstract— In the present paper, we analyzed double diffusive free convection past a vertical wavy surface embedded in a fluid saturated porous medium with variable properties. The Darcy law is assumed to describe the homogenous fluid saturated porous medium. The temperature dependent variable properties (variable viscosity and variable thermal conductivity) are considered. The fluid flow, momentum, energy and solutal governing equations are transformed into boundary layer non-dimensional nonlinear ordinary differential equations with specified transformation and then solved with numerical technique. The results are reported for various physical parameters; variable viscosity, variable thermal conductivity, thermal dispersion and solutal dispersion and amplitude of the wavy surface on hydrodynamic velocity, temperature and concentration distributions as well as rate of heat (Nusselt number) and mass (Sherwood number) transfers. The numerical results obtained in the present method compared with previously published results and found to be in good agreement.

Index Terms— Vertical Wavy Surface, Double Dispersion Effects, Variable Properties, Darcy Porous Media, Free convection.

1 INTRODUCTION

In recent years, the study of natural convective flow, heat and mass transfer in porous media has received considerable interest in the literature. The interest for such studies is motivated by grain storage insulations, nuclear waste disposal, oil extraction, ground water pollution, resin transfer modeling, dispersion of chemical contaminants through water – saturated soil, fibrous insulations, packed beds and geo thermal systems. Comprehensive reviewers of the convection through Darcy porous media have been reported by Nield and Bejan [1] and by Ingham and Pop [2]. Darcy's law states that the volume averaged velocity is proportional to the pressure gradient. The present study deals with free convective flow on a vertical wavy surface embedded in a saturated porous medium. Natural convection from wavy surfaces is a topic of fundamental importance in heat transfer devices, such as flat-plate solar collectors and flat-plate condensers in refrigerators. Mainly, roughness elements disturbs the flow and alters the rate of heat and mass transfer, this is type of irregularities mostly occur in manufacturing. At first, Rees and Pop [3] investigated free convection along a vertical wavy surface in a porous medium. Cheng [4] studied natural convection heat and mass transfer near a vertical wavy surface with constant wall temperature and concentration in a porous medium.

Recently, Shalini and Rathish Kumar [5] investigated the influence of variable heat flux on natural convection along a corrugated wall in porous media. Mohamed et.al [6] studied combined radiation and free convection from a vertical wavy surface embedded in porous media. Elgazery and Elazem [7] investigated the effects of variable properties on MHD unsteady natural convection heat and mass transfer over a vertical wavy surface. Rathish kumar and Krishan Murthy [8] analyzed Soret and Dufour effects on double diffusive free convection from a corrugated vertical surface in a non-Darcy porous medium. Neagu [9] analyzed free convective heat and mass transfer induced by a constant heat and mass fluxes vertical wavy wall in a non-Darcy double stratified porous medium. Narayana et.al. [10] studied double diffusive convection and cross diffusion effects on a horizontal wavy surface in a porous medium. Parveen and Alim [11] investigated Joule heating and MHD free convection flow along a vertical wavy surface with viscosity and thermal conductivity dependent on temperature.

The hydrodynamic mixing is called dispersion, which is the secondary effect of a porous medium on the fluid flow takes place in the result of mixing and recirculation of local fluid particles through tortuous paths formed by the porous medium solid particles. There has been renewed interest in studying double diffusive convection due to the effect of thermal and solutal dispersions; these are additional energy and concentration mass transport process. In certain thermal and solutal dispersion applications such as those involving oil

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reservoir and geothermal engineering applications such as ceramic processing, sensible heat storage beds and petroleum recovery etc., In view of the aforesaid applications, many authors have analyzed the effects of thermal and solutal dispersion on convective heat and mass transfer through porous media. Abbas et.al. [12] studied effects of thermal dispersion on free convection in a fluid saturated porous medium. Kairi and Murthy [13] considered the double dispersion effects to study mixed convection heat and mass transfer in a non-Newtonian fluid saturated non-Darcy porous medium. Pathak and Ghiaasiaan [14] investigated convective heat transfer and thermal dispersion during laminar pulsating flow in porous media. The effects of MHD and double dispersion on free convection in a non-Darcy porous medium saturated with power law fluid are investigated by Srinivasacharya et.al [15]. Ramreddy [16] has been studied the effects of double dispersion on convective flow over a cone.

In view of the above application, the authors are envisage to investigate free convection along a vertical wavy surface embedded in a fluid saturated porous medium with variable properties and double dispersion effects. The governing boundary equations for flow mass, momentum, energy and concentration are transformed into non-dimensional nonlinear ordinary differential equations by using appropriate transformation and then solved by using numerical method. The results are reported graphically for various physical parameters for flow velocity, temperature and concentration distributions as well as Nusselt number and Sherwood number. The present results are compared with previously existing results and obtained a very good agreement.

2 Formulation of the Problem

We consider the steady, two dimensional laminar, viscous incompressible fluid over a vertical wavy plate embedded in a saturated porous medium. The configuration of the model and coordinate system is shown in fig. 1. We assume that the wavy surface is given by

$$\bar{y} = \bar{\sigma}(\bar{x}) = \bar{a}\sin\left(\frac{\pi\bar{x}}{l}\right) \tag{1}$$

Where 'a' represents amplitude of the wavy surface and 'l' represents the characteristics of wavy length. The plate is maintained with constant temperature T_w and concentration C_w, which are higher than the ambient fluid temperature T_∞ and concentration C_∞. The Darcy law can be used to describe the porous medium. In addition, we consider thermal and solutal dispersion effects. In view of the above assumptions and invoking the boundary layer and Boussinesq approximations, the governing boundary layer equations for the conservation of mass, momentum, energy and concentrations are:

$$\frac{\partial\bar{u}}{\partial\bar{x}} + \frac{\partial\bar{v}}{\partial\bar{y}} = 0 \tag{2}$$

$$\frac{\partial}{\partial\bar{y}}\left(\frac{\mu}{K}\bar{u}\right) = \frac{\partial}{\partial\bar{x}}\left(\frac{\mu}{K}\bar{v}\right) \pm \rho g\left(\beta_t\frac{\partial T}{\partial\bar{y}} + \beta_c\frac{\partial C}{\partial\bar{y}}\right) \tag{3}$$

$$\bar{u}\frac{\partial T}{\partial\bar{x}} + \bar{v}\frac{\partial T}{\partial\bar{y}} = \frac{\partial}{\partial\bar{x}}\left(\alpha_x\frac{\partial T}{\partial\bar{x}}\right) + \frac{\partial}{\partial\bar{y}}\left(\alpha_y\frac{\partial T}{\partial\bar{y}}\right) \tag{4}$$

$$\bar{u}\frac{\partial C}{\partial\bar{x}} + \bar{v}\frac{\partial C}{\partial\bar{y}} = \frac{\partial}{\partial\bar{x}}\left(D_x\frac{\partial T}{\partial\bar{x}}\right) + \frac{\partial}{\partial\bar{y}}\left(D_y\frac{\partial T}{\partial\bar{y}}\right) \tag{5}$$

The corresponding boundary conditions are

$$\left.\begin{array}{l}\bar{u}=0, \bar{v}=0, T=T_w, C=C_w, \ at \quad \bar{y}=\bar{\sigma}(\bar{x})=\bar{a}\sin\left(\frac{\pi\bar{x}}{l}\right)\\ \bar{u}\to 0, \ T\to T_\infty, C\to C_\infty \quad as \quad \bar{y}\to\infty\end{array}\right\} \tag{6}$$

where $\bar{u}$, and $\bar{v}$ are velocity components in $\bar{x}$ and $\bar{y}$ directions respectively. μ is the kinematic viscosity, K is the permeability of the porous medium, ρ is the density of the fluid, β_t is the thermal expansion coefficient, β_c is the solutal expansion coefficient, g is the acceleration due to gravity, α_x, D_x and α_y, D_y are the effective thermal and solutal diffusivities respectively, have the contribution of both molecular diffusion and hydrodynamic dispersion, these can be described as (see Telles and Trevisan [17])

$$\left.\begin{array}{l}\alpha_x = \alpha + \gamma d\bar{v}, \quad \alpha_y = \alpha + \gamma d\bar{u}\\ D_x = D + \zeta d\bar{v}, \ D_y = D + \zeta d\bar{u}\end{array}\right\} \tag{7}$$

where α is the thermal conductivity, D is the molecular diffusivities, γ is the coefficient of thermal dispersion and ζ is the solutal dispersion. The fluid properties namely, viscosity and thermal conductivities are assumed to be vary as an inverse linear and linear function of the temperature respectively and these can be written as (see [18-20])

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\left(1+\delta\left(T-T_\infty\right)\right) \text{ or } \frac{1}{\mu} = b\left(T-T_r\right) \text{ and } \alpha = \alpha_o\left(1+E(T-T_\infty)\right)$$

where

$$b = \frac{\delta}{\mu_\infty}, \text{ and } T_r = T_\infty - \frac{1}{\delta}.$$

Both b and T_r are constants and their values depend on the reference state and the thermal property of the fluid i.e. δ, α_o is the thermal diffusivity at the wavy surface temperature T_w and E is a constant depending on the nature of the fluid. In general, b>0 for liquids and b<0 for gases. It is worth mentioning here that E is positive for fluids such as air and E is negative for fluids such as lubrication oils. θ_r, which is defined by

$$\theta_r = \frac{T_r - T_\infty}{T_w - T_\infty} = \frac{-1}{\delta(T_w - T_\infty)} \tag{8}$$

is constant. The parameter θ_r was first introduced by Ling and Dybbs [21] It is worth mentioning here that for $\delta \to 0$ (i.e. $\mu = \mu_\infty$ =constant) then $\theta_r \to \infty$, the effect of viscosity is negligible. The variable thermal conductivity can be written in the non-dimensional form (see [20]) as

$$\alpha = \alpha_o(1 + \beta\theta) \tag{9}$$

where $\beta = E(T_w - T_\infty)$ is the thermal conductivity parameter. The variation of β can be taken in the range $-0.1 \le \beta \le 0$ for lubrication oils, $0 \le \beta \le 0.12$ for water and $0 \le \beta \le 6$ for air. We define the stream function $\bar{\psi}$, which is to be satisfied the continuity equation (2) such that

$$\bar{u} = \frac{\partial \bar{\psi}}{\partial \bar{y}}, \quad \bar{v} = -\frac{\partial \bar{\psi}}{\partial x}$$

To convert the governing boundary layer equations in non-dimensional form, we introduce the following dimensionless variables

$$\begin{gathered} x = \frac{\bar{x}}{l}, \quad y = \frac{\bar{y}}{l}, \quad a = \frac{\bar{a}}{l}, \quad \sigma = \frac{\bar{\sigma}}{l}, \\ \psi^* = \frac{\bar{\psi}}{\alpha_0}, \quad \theta = \frac{T - T_\infty}{T_w - T_\infty}, \quad \varphi = \frac{C - C_\infty}{C_w - C_\infty} \end{gathered} \tag{10}$$

By using eqs. (7) – (10), the eqs. (3) – (5) reduces to

$$\frac{1}{\theta - \theta_r}\left(\frac{\partial \theta}{\partial y}\frac{\partial \psi^*}{\partial y} + \frac{\partial \theta}{\partial x}\frac{\partial \psi^*}{\partial x}\right) + \frac{\partial^2 \psi^*}{\partial y^2} + \frac{\partial^2 \psi^*}{\partial x^2} = Ra\left(1 - \frac{\theta}{\theta_r}\right)\left(\frac{\partial \theta}{\partial y} + N\frac{\partial \varphi}{\partial y}\right) \tag{11}$$

$$\begin{aligned} \frac{\partial \psi^*}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \psi^*}{\partial x}\frac{\partial \theta}{\partial y} &= \beta\left(\left(\frac{\partial \theta}{\partial x}\right)^2 + \left(\frac{\partial \theta}{\partial y}\right)^2\right) + (1 + \beta\theta)\left(\frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2}\right) \\ &+ \frac{\gamma d}{l}\left[\frac{\partial^2 \psi^*}{\partial y^2}\frac{\partial \theta}{\partial y} - \frac{\partial^2 \psi^*}{\partial x^2}\frac{\partial \theta}{\partial x} + \frac{\partial \psi^*}{\partial y}\frac{\partial^2 \theta}{\partial y^2} - \frac{\partial \psi^*}{\partial x}\frac{\partial^2 \theta}{\partial x^2}\right] \end{aligned} \tag{12}$$

$$\begin{aligned} Le\left(\frac{\partial \psi^*}{\partial y}\frac{\partial \varphi}{\partial x} - \frac{\partial \psi^*}{\partial x}\frac{\partial \varphi}{\partial y}\right) &= \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right) \\ &+ \frac{\zeta d}{l}\left[\frac{\partial^2 \psi^*}{\partial y^2}\frac{\partial \phi}{\partial y} - \frac{\partial^2 \psi^*}{\partial x^2}\frac{\partial \phi}{\partial x} + \frac{\partial \psi^*}{\partial y}\frac{\partial^2 \phi}{\partial y^2} - \frac{\partial \psi^*}{\partial x}\frac{\partial^2 \phi}{\partial x^2}\right] \end{aligned} \tag{13}$$

where $Ra = \frac{gK\beta_t(T_w - T_\infty)l}{\alpha_o \nu}$ is the modified - Rayleigh number, $\nu = \frac{\mu_\infty}{\rho}$ is the kinematic viscosity of the fluid, $N = \frac{\beta_c(C_w - C_\infty)}{\beta_t(T_w - T_\infty)}$ is the buoyancy ratio, $Le = \frac{\alpha_o}{D}$ is the Lewis number, and $Ra_d = \frac{gK\beta_t(T_w - T_\infty)d}{\alpha_o \nu}$ is the pore diameter dependent Rayleigh number which describes the relative intensity of the buoyancy force, such that d is the pore diameter.

The associated boundary conditions are given by

$$\left.\begin{aligned} &\psi^* = 0, \ \theta = 1, \ \varphi = 1, \ on \ \ y = a\sin(x), \\ &\psi^*_y \to 0, \ \theta \to 0 \ \ as \ \ y \to \infty, \end{aligned}\right\} \tag{14}$$

Let us consider the following transformations

$$x = \xi, \quad \eta = \frac{y - a\sin(x)}{\xi^{1/2} Ra_x^{-1/2}}, \quad \psi^* = Ra_x^{1/2}\psi. \tag{15}$$

Invoking the eq (15) and letting $Ra_x \to \infty$ into eqs. (11) - (14) reduces into the following boundary layer equations:

$$\begin{aligned} &\frac{1}{\theta - \theta_r}(1 + a^2\cos^2(\xi))\frac{\partial \theta}{\partial \eta}\frac{\partial \psi}{\partial \eta} + (1 + a^2\cos^2(\xi))\frac{\partial^2 \psi}{\partial \eta^2} \\ &\qquad = \xi^{1/2}\left(1 - \frac{\theta}{\theta_r}\right)\left(\frac{\partial \theta}{\partial \eta} + N\frac{\partial \psi}{\partial \eta}\right) \end{aligned} \tag{16}$$

$$\begin{aligned} &\xi^{1/2}\left(\frac{\partial \psi}{\partial \eta}\frac{\partial \theta}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \theta}{\partial \eta}\right) = (1 + a^2\cos^2(\xi))(1 + \beta\theta)\frac{\partial^2 \theta}{\partial \eta^2} \\ &+ \beta(1 + a^2\cos^2(\xi))\left(\frac{\partial \theta}{\partial \eta}\right)^2 + Ds\xi^{-1}(1 + a^2\cos^2(\xi))\left[\frac{\partial^2 \psi}{\partial \eta^2}\frac{\partial \theta}{\partial \eta} + \frac{\partial \psi}{\partial \eta}\frac{\partial^2 \theta}{\partial \eta^2}\right] \end{aligned} \tag{17}$$

$$\begin{aligned} &Le\xi^{1/2}\left(\frac{\partial \psi}{\partial \eta}\frac{\partial \varphi}{\partial \xi} - \frac{\partial \psi}{\partial \xi}\frac{\partial \varphi}{\partial \eta}\right) = (1 + a^2\cos^2(\xi))\frac{\partial^2 \varphi}{\partial \eta^2} \\ &\qquad + Dc(1 + a^2\cos^2(\xi))\left(\frac{\partial^2 \psi}{\partial \eta^2}\frac{\partial \phi}{\partial \eta} + \frac{\partial \psi}{\partial \eta}\frac{\partial^2 \phi}{\partial \eta^2}\right) \end{aligned} \tag{18}$$

where $Ds = \gamma.Ra_d$ is the thermal dispersion parameter $Dc = \zeta.Ra_d$ is the solutal dispersion parameter. To transform Eqs. (16) - (18) into a set of ordinary differential equations, we introduce the following similarity transformations

$$\hat{\eta} = \frac{\eta}{1 + a^2\cos^2(\xi)}, \quad \psi = \xi^{1/2} f(\hat{\eta}), \quad \theta = \theta(\hat{\eta}) \quad and \quad \varphi = \varphi(\hat{\eta}) \tag{19}$$

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Thus, we obtain

$$f'' + \frac{1}{\theta - \theta_r}\theta' f' = \left(1 - \frac{\theta}{\theta_r}\right)(\theta' + N\varphi') \tag{20}$$

$$\beta(\theta')^2 + (1+\beta\theta)\theta'' + \frac{1}{2} f\theta' + Ds\frac{(1+a^3\cos^3(\xi))}{(1+a^2\cos^2(\xi))^2}\left(f''\theta' + \theta'' f'\right) = 0 \tag{21}$$

$$\frac{1}{Le}\varphi'' + \frac{1}{2} f\varphi' + Dc\frac{(1+a^3\cos^3(\xi))}{(1+a^2\cos^2(\xi))^2}\left(f''\phi' + \phi'' f'\right) = 0 \tag{22}$$

where prime denotes differentiation with respect to $\hat{\eta}$.

The associated boundary conditions are

$$\begin{aligned} &f = 0, \quad \theta = 1, \quad and \quad \varphi = 1 \quad at \ \hat{\eta} = 0 \\ &f' \to 0, \quad \theta \to 0 \quad and \quad \varphi \to 0 \quad as \ \hat{\eta} \to \infty \end{aligned} \tag{23}$$

The engineering design quantities of physical interest include Nusselt number and Sherwood numbers which are defined as

$$\begin{aligned} Nu_\xi &= -\left(1 + Ds\frac{f'(0)}{(1+a^2\cos^2(\xi))}\right)\frac{\theta'(0)Ra_\xi^{1/2}}{(1+a^2\cos^2(\xi))^{1/2}} \\ and \quad Sh_\xi &= -\left(1 + Ds\frac{f'(0)}{(1+a^2\cos^2(\xi))}\right)\frac{\varphi'(0)Ra_\xi^{1/2}}{(1+a^2\cos^2(\xi))^{1/2}} \end{aligned} \tag{24}$$

3 Results and Discussion

The problem of free convection along a vertical wavy surface embedded in fluid saturated Darcy porous media subject to the variable viscosity, variable thermal conductivity and double dispersion effects has been investigated. A simple coordinate transformation is employed to reduce the governing non-linear boundary equations into non-linear ordinary differential equations and then employed Runge-Kutta method with shooting technique. We restrict the physical parameter values $1<\theta_r \le 5$, $1\le\beta\le5$, $0<Ds\le1$, $0<Dc\le1$, and $0.5\le\xi\le2$ with the fixed values N=1, Le=1 and a=0.5. In order to validate the present method the numerical results obtained using the Runge Kutta method with shooting method are compared with Cheng [22] results. Table-1 shows the comparison results in the absence of variable properties and double dispersion effects (i.e. Ds=0 and Dc=0) over vertical wavy surface with Cheng [22] and the results are found to be in good agreement.

We have found the numerical solutions for non dimensional velocity, temperature and concentration distributions as well as rate of heat and mass transfer coefficients as shown graphically in Figs. (2) – (24). The variation of variable viscosity parameter (θ_r) on non-dimensional velocity, temperature and concentration distributions is presented in figs. (2) – (4). It is noticed from fig. (2) that an increase in variable viscosity

Table-1: Comparison of the rate of heat and mass transfer for a=0, β=0, τ=0 and $\theta_r \to\infty$ at N=1, and Le=0.5.

		$Nu_x Ra_x^{-1/2}$	$Sh_x Ra_x^{-1/2}$	$Nu_x Ra_x^{-1/2}$	$Sh_x Ra_x^{-1/2}$
Le	**N**	**Cheng [22]**	**Cheng [22]**	**Present**	**Present**
1	4	0.992	0.992	0.9923	0.9923
10	4	0.681	3.290	0.6809	3.2883
100	4	0.521	10.521	0.5208	10.5205
4	1	0.559	1.358	0.5558	1.3565
4	2	0.650	1.624	0.6510	1.6238
4	3	0.728	1.852	0.7275	1.8532

parameter θ_r resulted in depreciation in velocity distribution near the plate up to reach certain value and then increase the velocity profile until approaches a constant value (zero) at outer boundary layer regime. From figs. (3) and (4) we conclude that increasing variable viscosity parameter θ_r, clearly substantially enhances the temperature and concentration distributions.

The variation of variable thermal conductivity (β) on velocity, temperature and concentration distributions is illustrated in figs. (5) – (7). The velocity profile results for different values of β are given by fig. (5), these results are having similar behavior as shown in fig. (2). From fig. (6) it is evident that temperature profiles is more pronounced with increasing values of β. Conversely, a strong decrease in concentration distribution as shown in Fig. (7); occurs with increasing values of β.

The effect of thermal dispersion parameter (Ds) on the non-dimensional velocity, temperature and concentration is depicted in figs. (8) – (10). From fig. (8) we conclude that the results of velocity profile reduce near the surface for larger values of thermal dispersion parameter and opposite results are observed as the radial distance moves far away from the surface with increase in thermal dispersion parameter. The presence of thermal dispersion in the energy equation gives thermal conduction more dominance. It is observed from fig. (9) that increasing thermal dispersion parameter tends to enhance the temperature distribution. i.e. thermal dispersion enhances the transport of heat along radial direction to the plate. It is noticed from fig. (10) that the solutal boundary layer thickness is reduced as increase in thermal dispersion parameter.

The set of figs. (11) – (13) are plotted for the variation of non-dimensional velocity, temperature and concentration distributions across the boundary layer for different values of solutal dispersion parameter (Dc). From fig. (11) we noticed that an increase in Dc is seen to significantly enhance the momentum boundary thickness. It is observed from fig. (12) that temperature profile reduced with increase in Dc. It can be evident from this figure that as Dc increases thermal boundary layer thickness increases. It is observed from fig. (13) that increasing the solutal dispersion parameter (Dc), accelerates the concentration of the fluid. Hence the concentration boundary layer thickness increases with an increase in solutal dispersion parameter (Dc).

Figs. (14) – (16) illustrate the velocity, temperature and concentration distributions for different values of ξ-location. It can be found from fig. (14) that velocity profile is increased with increase in ξ-location. Hence the hydrodynamic boundary layer thickness increases as increase in ξ-location. We noticed from figs. (15) and (16) that the similar behavior of temperature and concentration profile, in comparison with velocity distribution as shown in fig. (14). It is an important to note that it quickly reaches similarity solutions not far away from the leading edge.

The variation of rate of heat and mass transfer (Nusselt number and Sherwood number) with streamwise coordinate at the wall are shown in figs. (17) – (18) for different values of variable viscosity parameter (θ_r). It is noticed from these figures that both Nusselt number and Sherwood number decreases with increase in θ_r. Hence, it is clear that increase in θr results an depreciation in the amplitude of the Nusselt number and Sherwood. Figs (19) – (20) represent the variation of Nusselt number and Sherwood number for different values of variable thermal conductivity parameter (β). Figs. (19) – (20) demonstrates that Nusselt number and Sherwood number reduces considerably for larger values of β. Figs. (21) – (22) reveals that enhancement of thermal dispersion parameter results enhancement in the amplitude of the Nusselt number and Sherwood number. The variation of Nusselt number and Sherwood number for different values of solutal dispersion parameter (Dc) is given in figs. (23) – (24). Figs. (23) – (24) exhibits the similar behavior of Nusselt number and Sherwood with streamwise coordinate, in comparison with what observed in figs. (21) – (22). Figs. (25) – (26) illustrates the variation of Nusselt number and Sherwood number for different values of the amplitude of the wavy surface. For 'a = 0' the vertical wavy surface reduces to vertical flat surface. It is noticed from figs. (25) and (26) that increasing the amplitude of the wavy surface, clearly substantially enhances the amplitude of the Nusselt number and Sherwood number with streawise coordinate.

3 CONCLUSSIONS

The effect of thermal and solutal dispersion on free convection along a vertical wavy surface with temperature dependent viscosity and thermal conductivity analyzed and the governing boundary non-dimensional equations are solved by employing Runge-Kutta method with shooting technique. The main results in this study are as follows:

1. Increasing variable viscosity parameter leads to decrease velocity distribution, Nusselt number and Sherwood number while temperature and concentration distributions are increases.
2. Increasing thermal dispersion parameter tends to increase velocity and temperature distributions as well as Nusselt number and Sherwood number while opposite results are noticed for concentration profile.
3. Velocity and concentration distributions as well as Nusselt number and Sherwood number are increased with increase in solutal dispersion parameter while we noticed that opposite results are reported for temperature profile.
4. Increase in the amplitude of the wavy surface results an enhancement in Nusselt number and Sherwood number.

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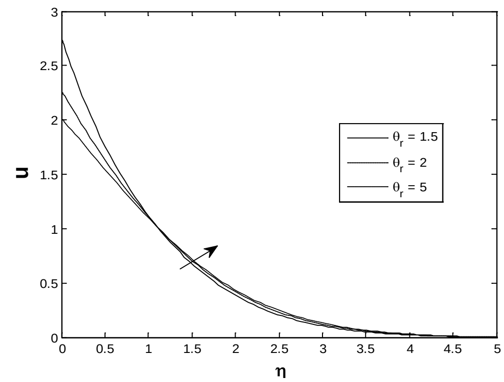

Fig.1. Velocity profile for different values of variable viscosity parameter for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5,β=0.5, and ξ=1.

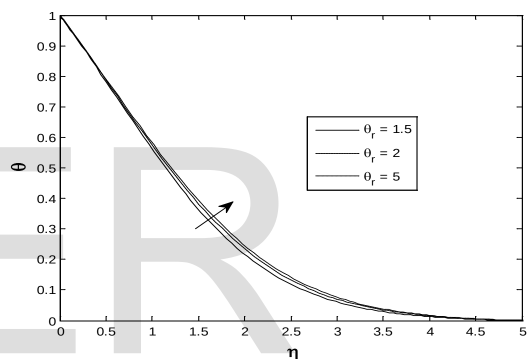

Fig.2. Temperature profile for different values of variable viscosity parameter for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, β=0.5, and ξ=1.

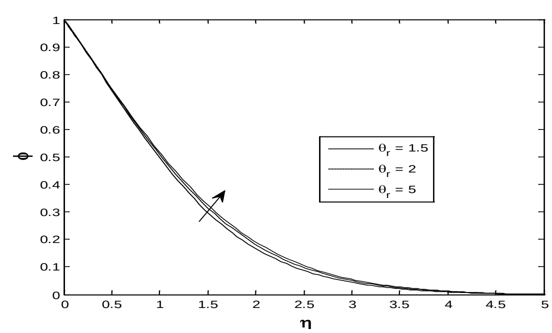

Fig.3. Concentration profile for different values of variable viscosity parameter for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, β=0.5, and ξ=1.

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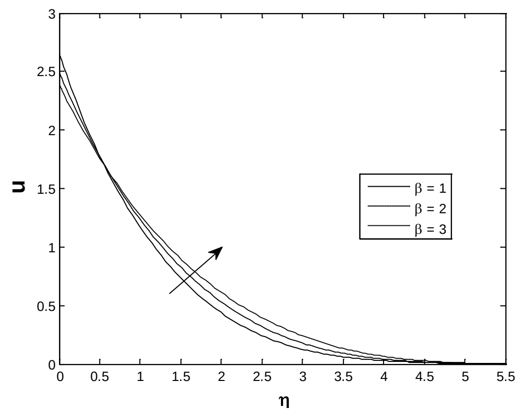

Fig.5. Velocity profile for different values of variable thermal conductivity parameter for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, θ_r=1.5, and ξ=1.

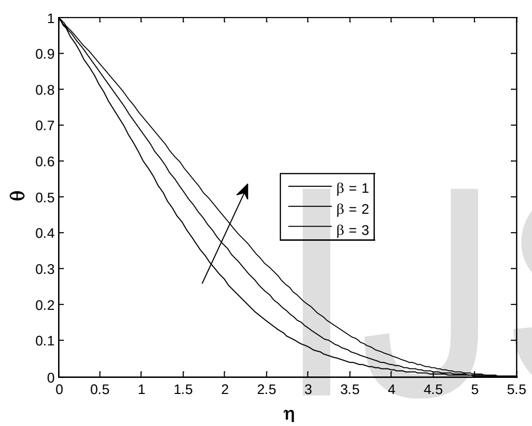

Fig.6. Temperature profile for different values of variable thermal conductivity parameter for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, θ_r=1.5, and ξ=1.

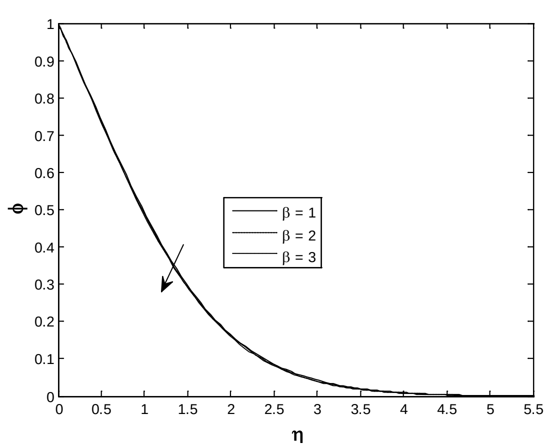

Fig.7. Concentration profile for different values of variable thermal conductivity parameter for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, θ_r=1.5, and ξ=1.

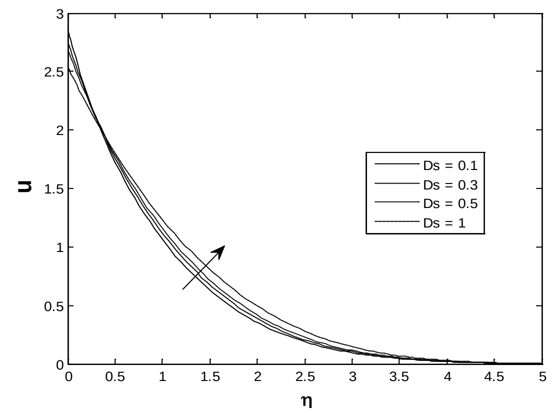

Fig.8. Velocity profile for different values of Ds for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, θ_r=1.5, β=0.5, and ξ=1.

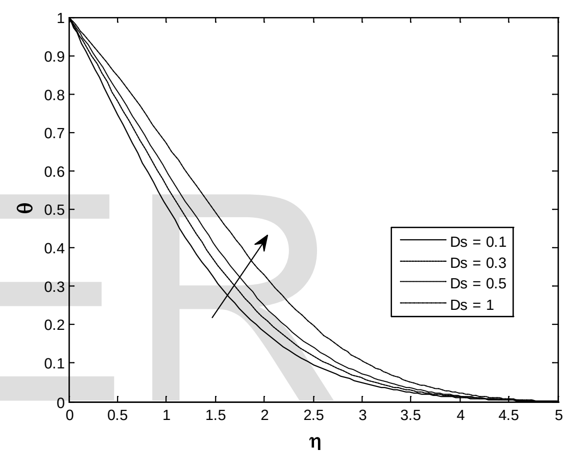

Fig.9. Temperature profile for different values of Ds for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, θ_r=1.5, β=0.5, and ξ=1.

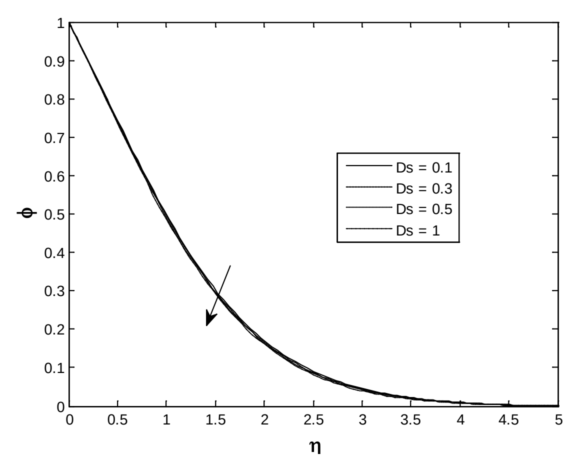

Fig.10. Concentration profile for different values of Ds for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, θ_r=1.5, β=0.5, and ξ=1.

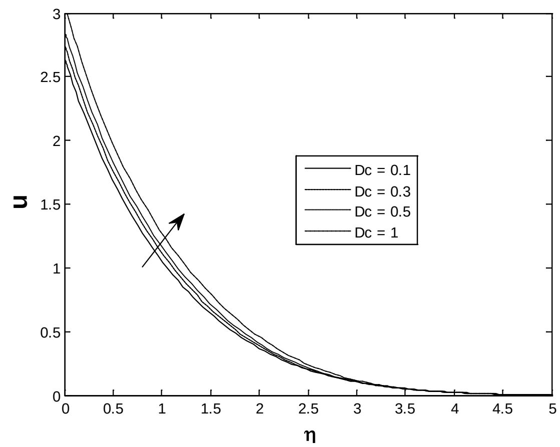

Fig.11. Velocity profile for different values of Dc for N=1, Le=1, Ds=0.3, a=0.5, θ_r=1.5, β=0.5, and ξ=1.

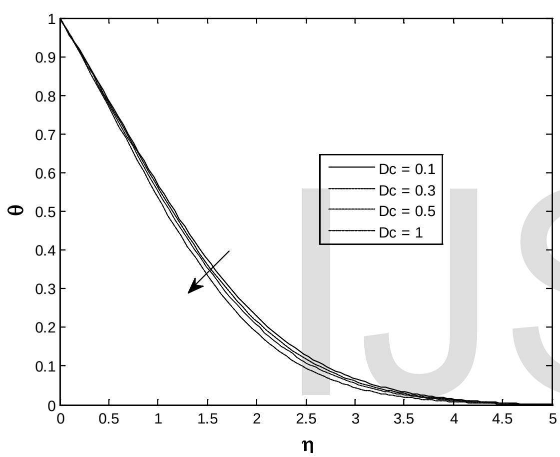

Fig.12. Temperature profile for different values of Dc for N=1, Le=1, Ds=0.3, a=0.5, θ_r=1.5, β=0.5, and ξ=1.

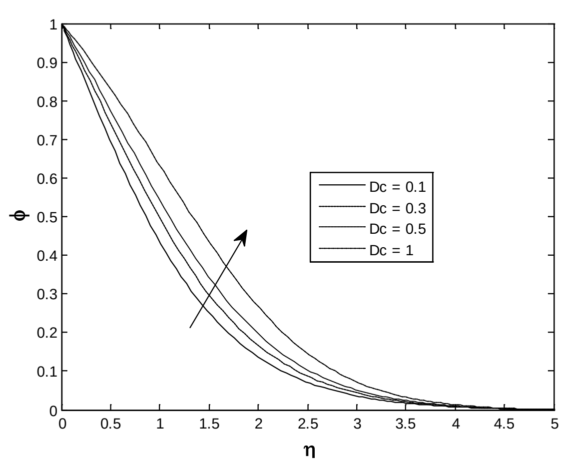

Fig.13. Concentration profile for different values of Dc for N=1, Le=1, Ds=0.3, a=0.5, θ_r=1.5, β=0.5, and ξ=1.

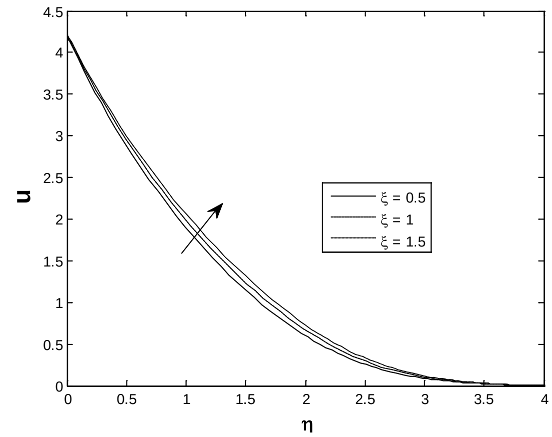

Fig.14. Velocity profile for different values of Dc for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, θ_r=1.5, β=0.5.

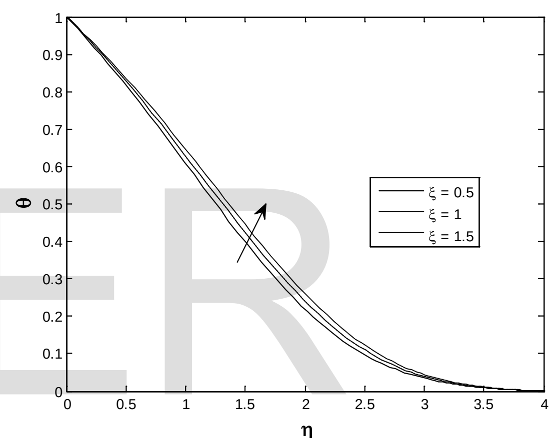

Fig.15. Temperature profile for different values of Dc for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, θ_r=1.5, β=0.5.

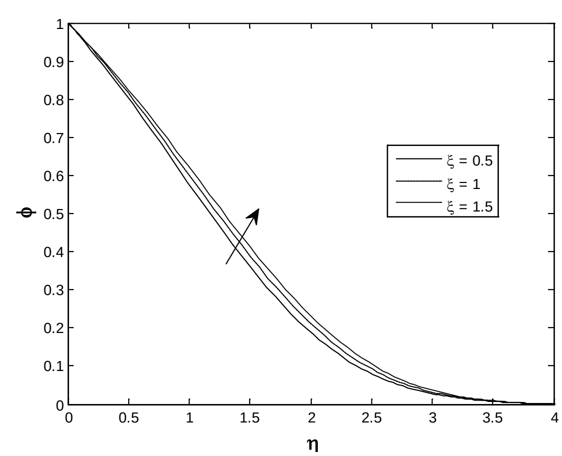

Fig.16. Concentration profile for different values of Dc for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, θ_r=1.5, β=0.5.

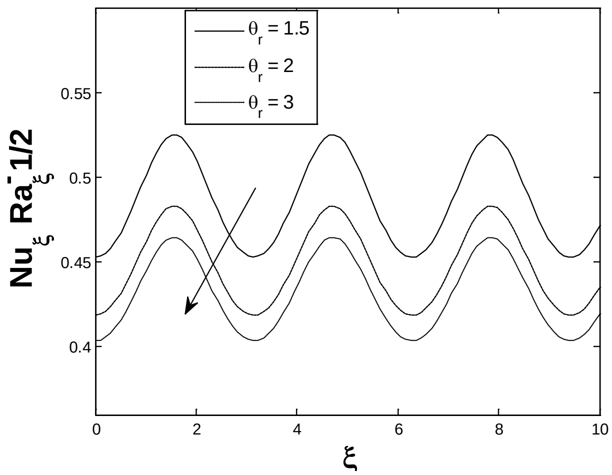

Fig.17. Axial distribution of the Nusselt number for different values of variable viscosity for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, ξ=1, β=0.5.

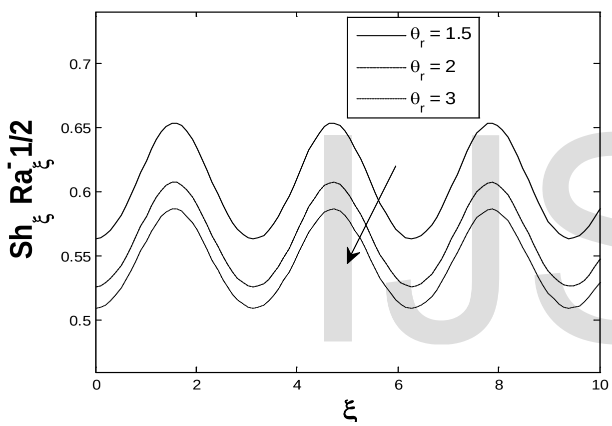

Fig.18. Axial distribution of the Sherwood number for different values of variable viscosity for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, ξ=1, β=0.5.

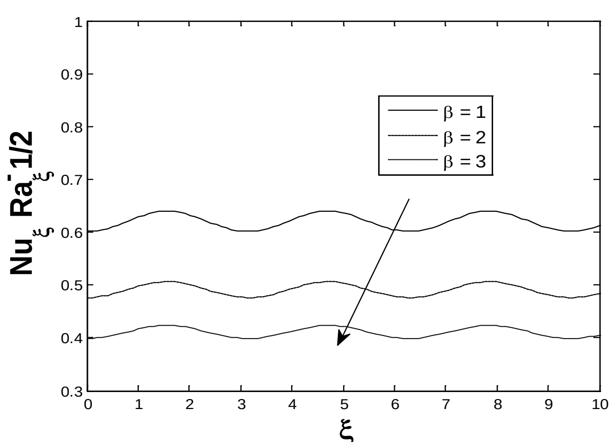

Fig.19. Axial distribution of the Nusselt number for different values of variable thermal conductivity for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, ξ=1, θ_r=1.5.

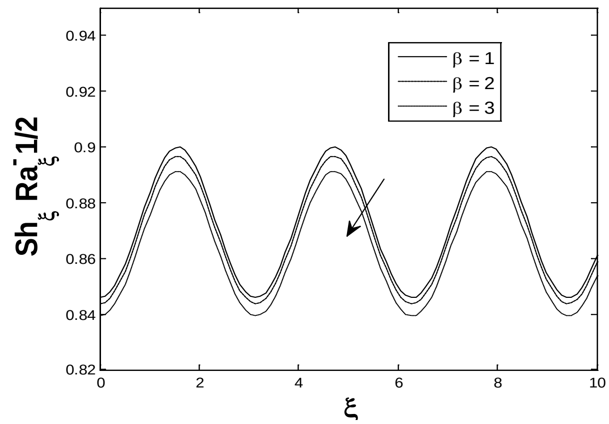

Fig.20. Axial distribution of the Sherwood number for different values of variable thermal conductivity for N=1, Le=1, Ds=0.3, Dc=0.3, a=0.5, ξ=1, θ_r=1.5.

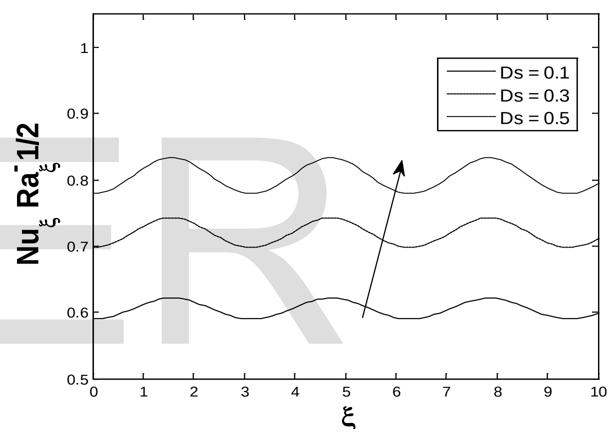

Fig.21. Axial distribution of the Nusselt number for different values of Ds for N=1, Le=1, β=0.5, Dc=0.3, a=0.5, ξ=1, θ_r=1.5.

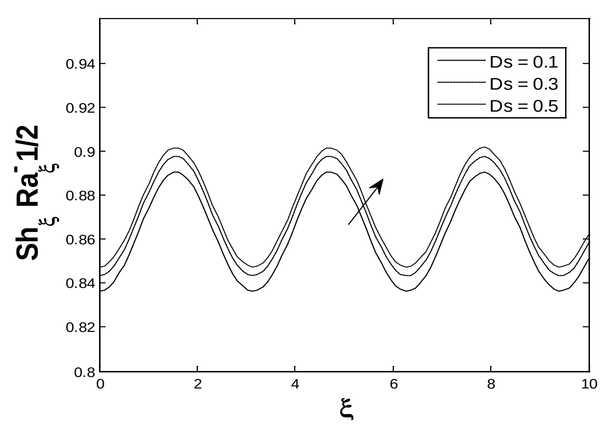

Fig.22. Axial distribution of the Sherwood number for different values of Ds for N=1, Le=1, β=0.5, Dc=0.3, a=0.5, ξ=1, θ_r=1.5.

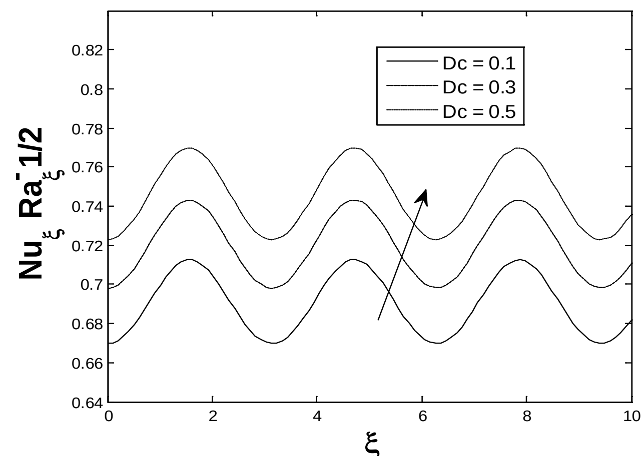

Fig.23. Axial distribution of the Nusselt number for different values of Dc for N=1, Le=1, β=0.5, Ds=0.3, a=0.5, ξ=1, θ_r=1.5.

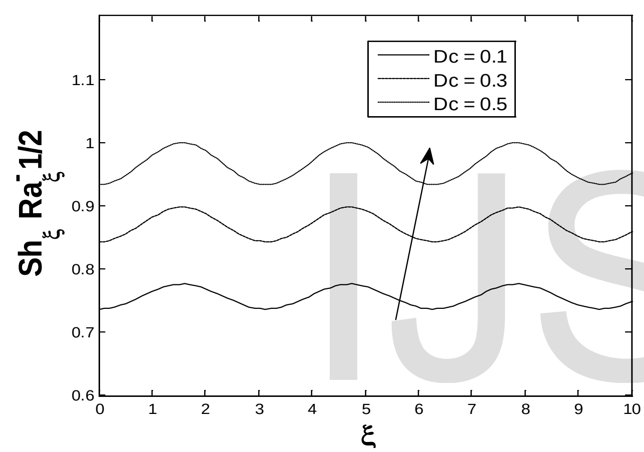

Fig.24. Axial distribution of the Sherwood number for different values of Dc for N=1, Le=1, β=0.5, Ds=0.3, a=0.5, ξ=1, θ_r=1.5.

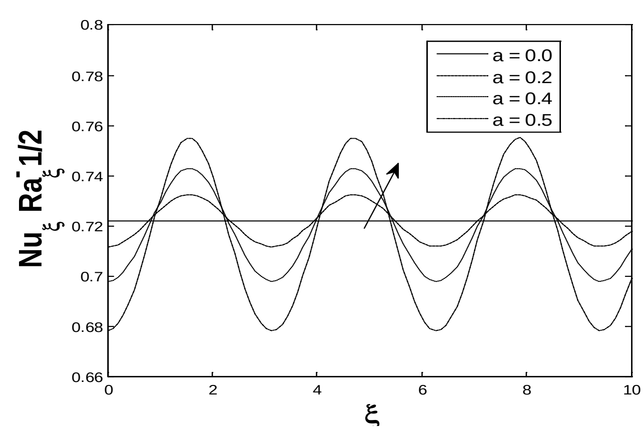

Fig.25. Axial distribution of the Nusselt number for different values of 'a' for N=1, Le=1, β=0.5, Ds, =0.3, Dc=0.3, a=0.5, ξ=1, θ_r=1.5.

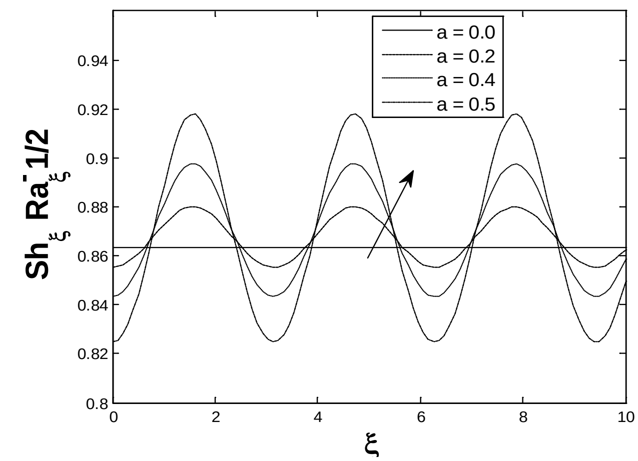

Fig.26. Axial distribution of the Sherwood number for different values of 'a' for N=1, Le=1, β=0.5, Ds, =0.3, Dc=0.3, a=0.5, ξ=1, θ_r=1.5.